Optics Lab

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Physics 480L

I. ABSTRACT

This lab explored several optical phenomena. A laser was used as the light source and a detector connected to an oscilloscope was used to find the intensities of various components of the light. The index of refraction of BK7 was measured to be 1.52±0.01. The phase shifts due to reflection off a quarter wave plate and a metal mirror were also measured. The phase shifts were 89.7±0.5° and 90.3±0.5°, respectively, and agreed with the theoretical values of 90° and 90°. The extinction ratio of two linear polarizers was calculated. The experimentally determined value was 43800±400. Using two linear polarizers, Malus' law was verified by plotting and fitting data. The same data was also used to verify the linearity of the detector. The position of the c-axis in a sample of sapphire was also found experimentally. The value was 28.3±0.2°.

II. INTRODUCTION

When light is incident on a surface, some of it is reflected and some of it is transmitted. The angle at which all of the light is transmitted is called the Brewster Angle, θ_B. θ_B is given by the equation, $\tan\theta_B = n_2/n_1$, where light travels from a material with index of refraction n_1 to a material with index of refraction n_2.

Linearly polarized light can undergo a phase shift when reflected from a material. Using Jones Vectors, the incoming electromagnetic wave can be represented as $\mathbf{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$. A reflection can be modeled as a transformation of the Jones Vector. One component **E** is retarded by a certain amount and a phase shift is introduced. Thus the Jones Vector becomes $\begin{bmatrix} a \\ be^{i\sigma} \end{bmatrix}$. The intensity, I, is the square of the electric field.

When light passes through two linear polarizers, the intensity of the outgoing light is governed by Malus' Law: $I = I_o \cos^2 \theta$, where θ is the angle between the axes of the polarizers.

Birefringent materials have two different indices of refraction. Anisotropies in the crystalline forces of the material cause the different indices. If the y-axis and z-axis have crystalline forces that are equal, then the x-axis is often called the optic axis. Electromagnetic waves oriented perpendicular to the optic axis are called ordinary and waves experience the ordinary index of refraction, n_o. Waves oriented parallel to the optic axis are called extraordinary and experience the extraordinary index of refraction, n_e. The value of n_o-n_e for sapphire is .00806. [(1)]

III. APPARATUS

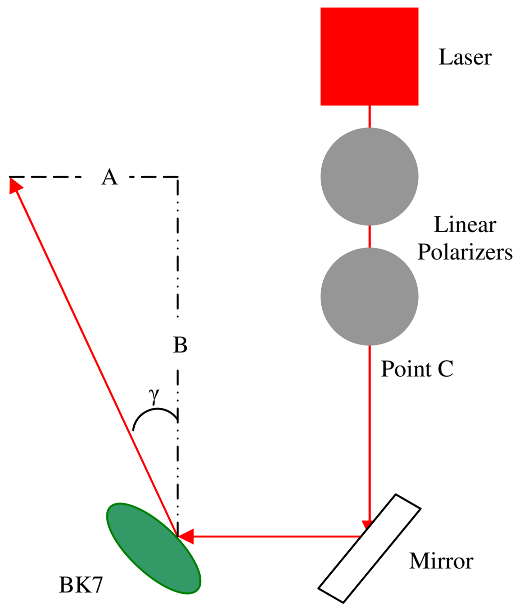

Figure 1: Experimental setup for finding the Brewster Angle for BK7.

The apparatus shown in Figures 1-3 were used to take data. A laser emitted light that was passed through two linear polarizers. Using these two polarizers, the phase and intensity of light passing Point A (in the Figures) could be controlled. The light passing Point A was reflected from a mirror (or wave plate). The output of the detector in Figures 2 and 3 was read by an oscilloscope.

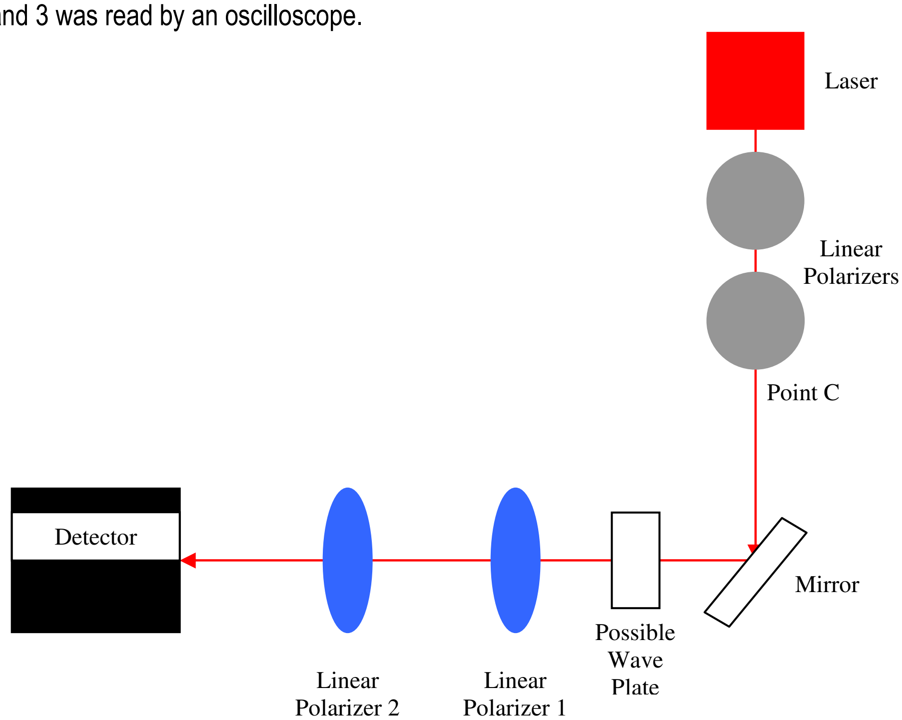

Figure 2: Experimental setup for measuring phase shift and Malus' Law.

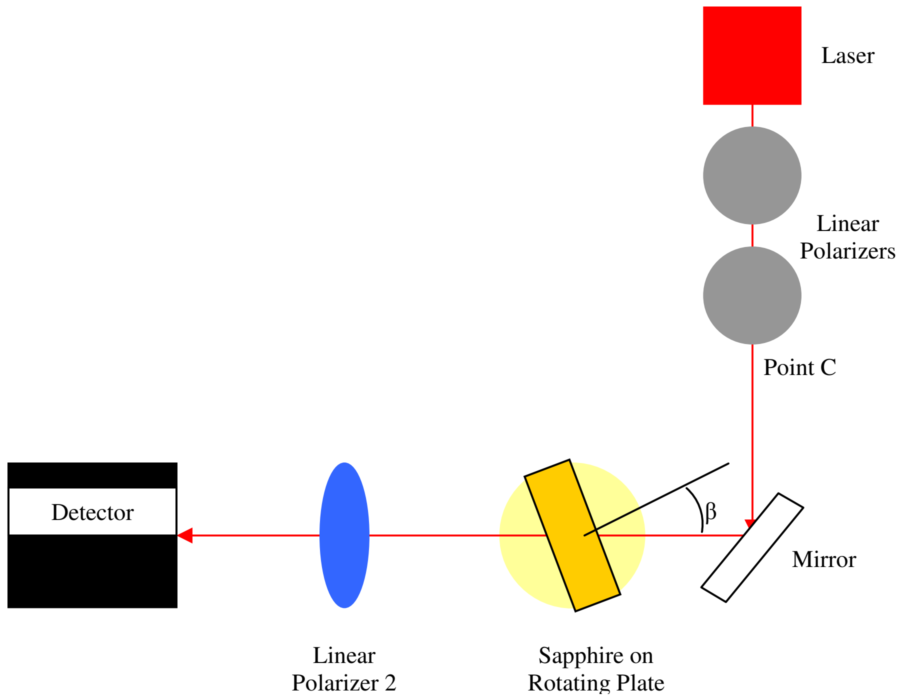

Figure 3: Experimental setup for the birefringence of sapphire.

IV. PROCEDURE

The experimental setup shown in Figure 1 was used to find the Brewster Angle of BK7. Horizontally polarized light passed by Point C. The lengths of A and B (as labeled in Figure 1) were measured. This gave a value for γ, which was used to calculate the Brewster Angle.

The setup similar to the one shown in Figure 2, using Linear Polarizer 1 but not Linear Polarizer 2, was then used to determine the phase shift of linearly polarized light upon reflection from a metal mirror and a quarter wave plate. Light polarized at 45^{o} passed by Point C and reflected off of the mirror/wave plate. The reflected light passed through Linear Polarizer 1 before going to the detector. Measurements were taken when horizontally polarized light, vertically polarized light, and light polarized at 45^{o} were allowed to pass through Linear Polarizer 1.

The extinction ratio of two linear polarizers was determined with the setup shown in Figure 2. Horizontally polarized light was passed through Linear Polarizer 1 and 2 when they were both at 45^{o} and when Linear Polarizer 1 was at 45^{o} and Linear Polarizer 2 was at -45^{o}. Data was taken at each of these points.

The setup in Figure 2 was also used to test Malus' Law and the linearity of the detector. Horizontally polarized light was passed through Linear Polarizer 1 and 2. Linear Polarizer 2 was set to transmit horizontally polarized light. Linear Polarizer 1 was then rotated and data was taken. The intensity of light on the detector was determined as a function of the angle of Linear Polarizer 1.

The angle of the crystal c-axis of sapphire, α, was determined with the setup shown in Figure 3. Vertically polarized light was reflected off of the mirror and passed through the sapphire and Linear Polarizer 2. The sapphire was rotated to various angles where the intensity was a minimum. At each angle, the intensity of the vertical and horizontal components of the light passing through the sapphire was determined. This was accomplished by rotating Linear Polarizer 2 so that it transmitted vertically and horizontally polarized light.

V. CALCULATIONS

The value of θ_B for BK7 was calculated using the measured lengths of A and B (in Figure 1), which were 31.6±.5cm and 73.2±0.5cm, respectively. By geometry, $\tan^{-1}(A/B)=\gamma$ and $90^o=2\theta_B-\gamma$. From this θ_B was calculated to be 56.69^o. Because $\tan\theta_B=(n_2/n_1)$ and n_1 is the index of refraction of air (about 1) $n_2=1.52$. The error in n_2 comes from the equation $\Delta n_2 = \frac{1}{\cos^2\theta_B}\frac{1}{1+\left(\frac{A}{B}\right)^2}\sqrt{\left(\frac{A\Delta B}{B^2}\right)^2+\left(\frac{\Delta A}{B}\right)^2}$ and is .01. Thus the index of refraction of BK7 is 1.52±0.01.

As stated earlier, the Jones Vector for linearly polarized light reflected from a surface can be represented as $\mathbf{E}=\begin{bmatrix} a \\ be^{i\sigma}\end{bmatrix}$. If the electric field is broken into vertical and horizontal components, the measured value for the vertical component of the light, I_v, is $|a|^2$ and the horizontal component of the light, I_h, is $|b|^2$. The light measured at 45^o has an electric field equal to the vector sum of a and $be^{i\delta}$. Thus the intensity light measured at 45^o, $I(45^o)$, is given by $I(45^\circ)=\frac{1}{2}\left(|a|^2+|b|^2+2|a||b|\cos\sigma\right)$. The phase shift, δ, can be found from this and the error in δ is given by

$$\Delta\sigma = \frac{1}{\sqrt{1-\left[\frac{I(45^\circ)-\frac{a^2+b^2}{2}}{2ab}\right]}}\sqrt{\left(\frac{\Delta I(45^\circ)}{2ab}\right)^2+\left(\frac{1+\frac{a^2}{2}}{2b}\Delta a\right)^2+\left(\frac{1+\frac{b^2}{2}}{2a}\Delta b\right)^2}$$

. The phase sift of the mirror is 90.3 ± 0.5^o and the phase sift of the wave plate is $89.7\pm.5^o$.

The extinction ratio of two linear polarizers, a^2/ε^2, is found from the equation $\frac{I_{\|}}{I_{\perp}}=\frac{r^2a^2}{2\varepsilon^2}$, where $I_{\|}$ is the intensity when there is no angle between the polarizers, $I_{\perp}$ is the intensity when the angle between the polarizers is 90^o, and $r=\frac{n^2\cos\theta-\sqrt{n^2-\sin^2\theta}}{n^2\cos\theta+\sqrt{n^2-\sin^2\theta}}$. Using n=1.515 and $\theta=45^o$, $a^2/\varepsilon^2=43800$. The error in the extinction ratio is $\frac{2}{r}\sqrt{\left(\frac{\Delta I_{\|}}{I_{\perp}}\right)^2+\left(\frac{I_{\|}\Delta I_{\perp}}{I_{\perp}^2}\right)^2}$. Thus, the extinction ratio is 43800 ±400.

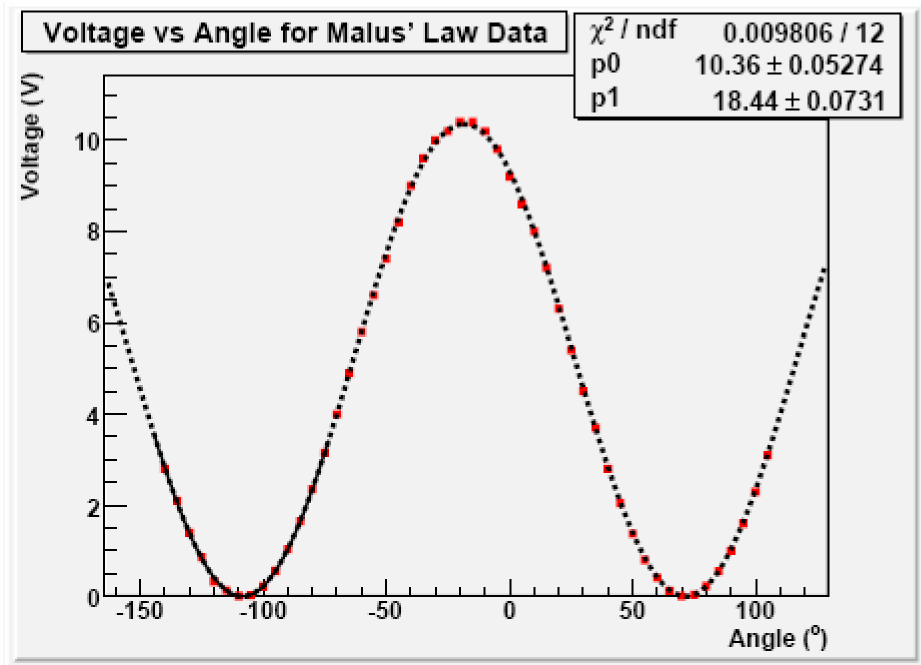

Figure 4: Plot to verify Malus' Law. Intensity at different θ between Linear Polarizer 1 and 2, equation=10.36cos^2(θ+18.44°).

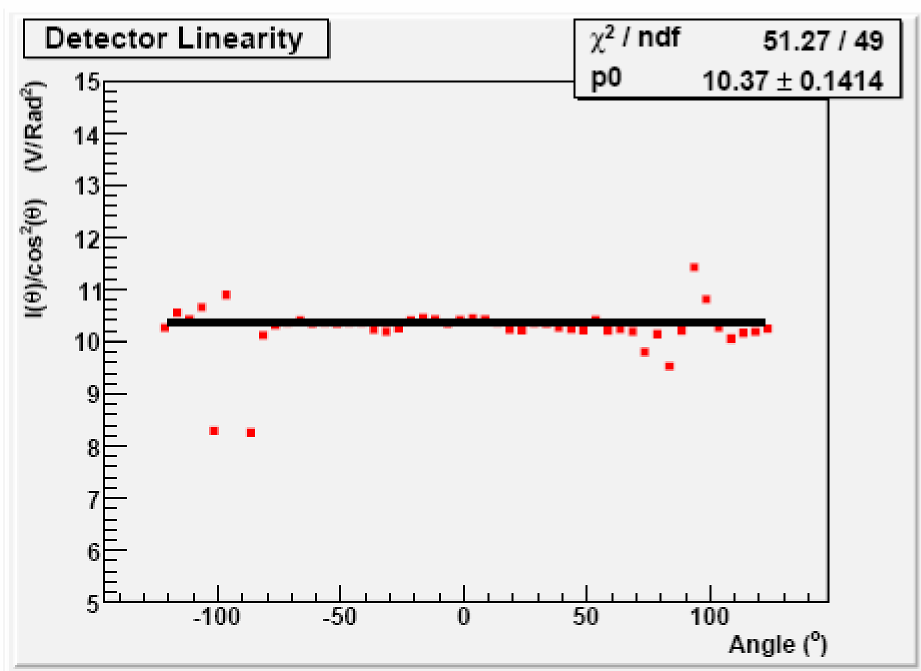

Figure 5: Plot of I/cos^{2}θ vs θ to verify detector linearity fit to a constant=10.37.

Figure 4 shows a plot of the intensity for different θ between the linear polarizers (1 and 2). In accordance with Malus' Law, the data was fitted to a cos^{2}θ equation. Figure 5 shows a plot of I/cos^{2}θ for different values of θ. The data was fitted to a constant.

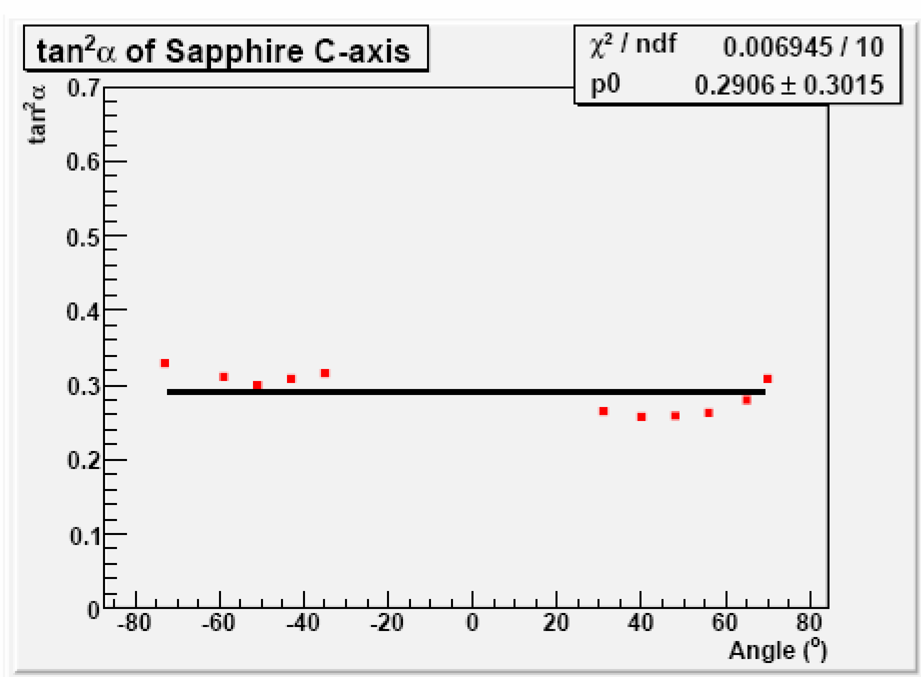

Figure 6: Plot of $\tan^2\alpha$ vs angle β fit to a constant=.291.

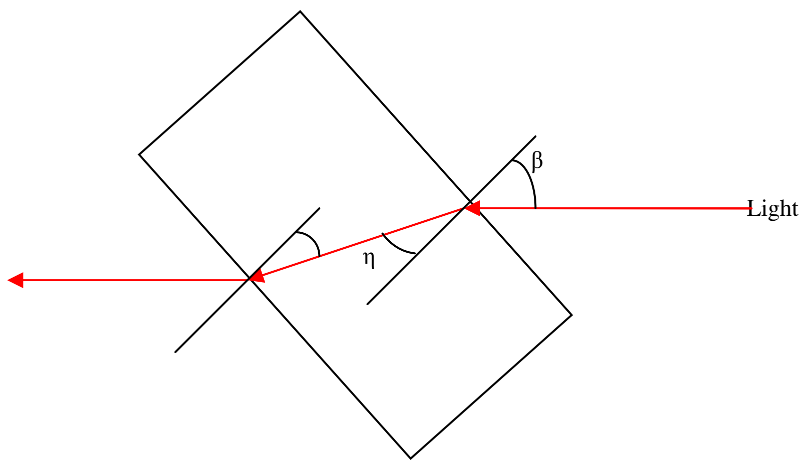

Figure 7: Explanation of angles used in finding α for sapphire.

The angle of the c-axis in sapphire, α, is determined by $\sin^2\psi = \frac{1}{\sqrt{1+\frac{\tan^2\alpha}{\sin^2\eta}}}$. Ψ is found by using the Fresnel Equations for the transmission coefficients: $t_\perp = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t}$ and $t_\parallel = \frac{2n_i\cos\theta_i}{n_i\cos\theta_{ti} + n_t\cos\theta_i}$ are used to correct the values for I_v and I_h. The intensities are then used to find the angle ψ. η if found by using Snell's law with the incident angle β and the index of refraction of sapphire (1.72). $\tan^2\alpha$ is plotted in Figure 6 and fitted to a constant. Thus, $\alpha=28.33^{o}$. The error in α is given by the equation $\Delta\alpha = \frac{\Delta k}{2\sqrt{k}(k+1)}$, where k is the constant fitted to $\tan^2\alpha$. Thus the error is 0.2^{o} and $\alpha=28.3\pm0.2^{o}$.

VI. CONCLUSIONS

Theoretically, the index of refraction of BK7 is 1.515. This agrees with the experimentally determined value of 1.52±0.01.

The phase shift due to a reflection off of a quarter wave plate is expected to be 90^{o}, which agrees with the experimental value of 89.7 ± 0.5^{o}. The phase shift of the mirror was determined to be 90.3 ± 0.5^{o}. This agrees with the theoretical value of 90^{o}.

The extinction ratio of the two linear polarizers is 43800 ±400.

The data taken for the intensity of light passing through two linear polarizers as a function of the angle between the polarizers verifies Malus' Law. The low χ^2/ndf for the plot in Figure 4 shows a good fit of the data to Malus' Law. Figure 5 shows that the detector is mostly linear.

The value for the position of the c-axis in sapphire is found to be 28.3 ± 0.2^{o}.

VII. References

1. Hecht, Eugene. *Optics*. Addison Wesley, Massachusetts; 1998.