Muon Decay Lifetime

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I. Abstract

The purpose of this experiment was to determine the muon lifetime and measure muon flux as a function of solid angle and zenith angle. The muon lifetime was measured with PMTs and a TAC connected to a computer. The lifetime was determined to be of 2.18±0.23μs, which agrees with the known value of 2.197μs [1]. The muon flux as a function of solid angle and zenith angle agrees with theoretical expectations.

II. Introduction

Muons are created from the decay of pions at high altitude in the atmosphere. Muons are unstable and decay into an electron and two neutrinos. Although their lifetimes are only 2.197μs, muons with high velocity can travel from high altitudes to the Earth's surface because of the time dilation of special relativity. The flux of muons at sea-level is known to be 1 muon/cm^2/min/steradian. This flux is also known to have a zenith angle dependence of $\cos^2\theta$. The number of decays as a function of time, $N(t)=N_0e^{-t/\tau}$, where N_0 is a constant and τ is the lifetime of the muon. [1]

III. Apparatus

The setup shown in Figure 1 was used in this experiment. The scintillators for PMTA and PMTD were squares 25.5cm on a side and the material for PMTB was a smaller square 10.5cm on a side. PMTC was a large block scintillator. All PMTs were fixed within a frame. PMTA had an adjustable height and was the only moveable PMT.

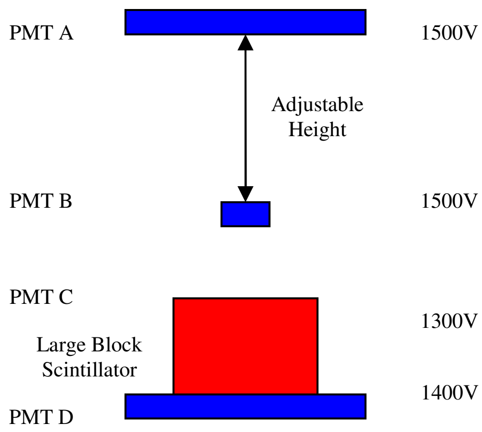

Figure 1: Experimental setup for PMTs

IV. Procedure

Counters PMTA and PMTB were connected to the electronic setup shown in Figure 2. The flux of muons was determined as the distance between PMTA and PMTB was varied (corresponding to various solid angles).

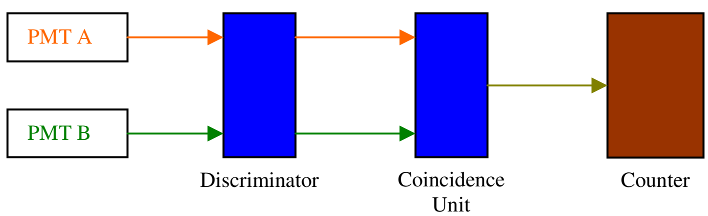

Figure 2: Setup used to determine muon flux at various solid angles and zenith angles

All four PMTs were then connected to the electronic setup shown in Figure 3, which was used to determine the muon lifetime. PMTA and PMTB were used in coincidence to determine the start time and PMTC was delayed and used to determine the stop time. PMTD acted as a veto to eliminate events from through-going muons. Two trials were done under the same conditions. The data from both trials was combined into a single data set for analysis.

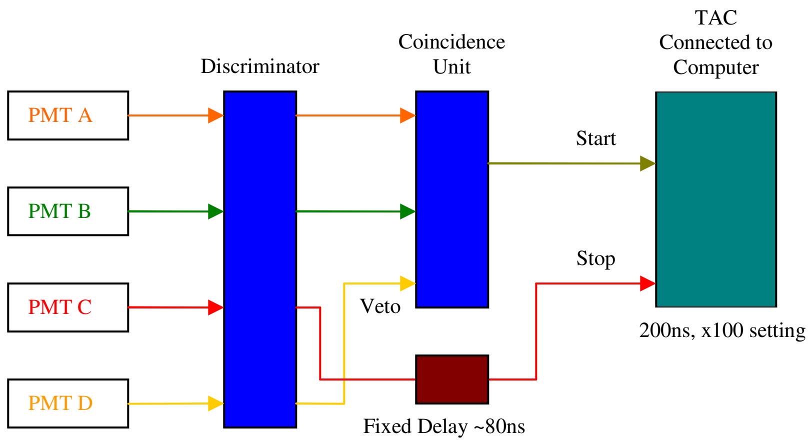

Figure 3: Setup used to determine muon decay lifetime

The setup shown in Figure 2 was then used to determine the flux of muons at various zenith angles. PMTA and PMTB were kept at a fixed separation, rotated at various angles, and the results were used to determine the flux as a function of zenith angle.

V. Results

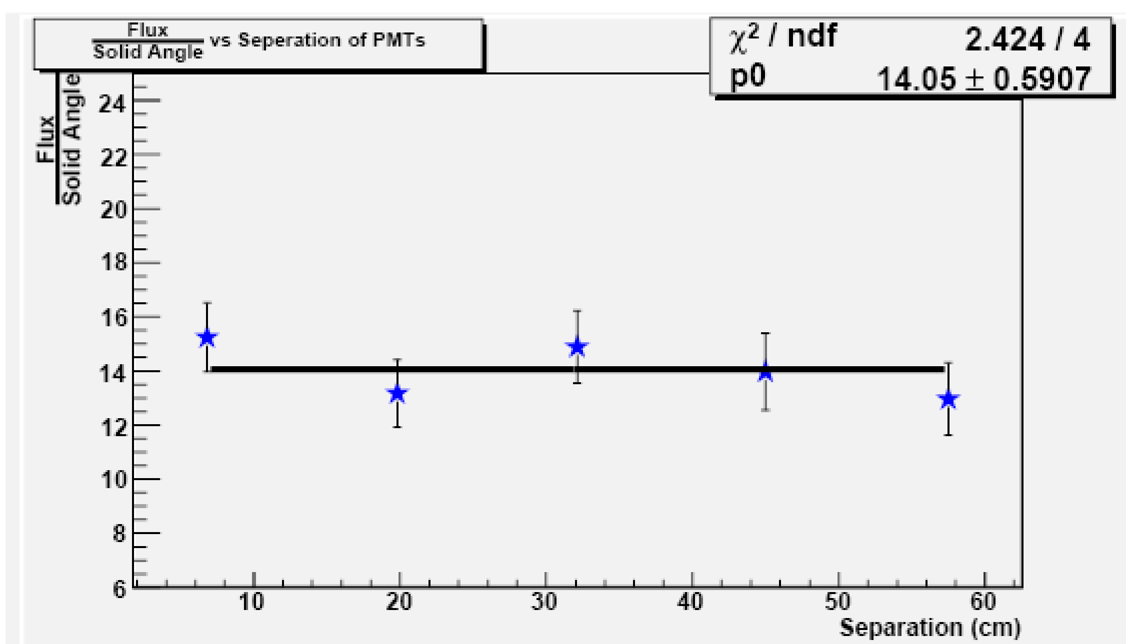

Figure 4: Plot of muon flux divided by solid angle versus separation fitted to a constant = 14.05

The data for the muon flux at different solid angles is shown in Figure 4. Because the flux of muons is dependent on the zenith angle, the calculation for the solid angle took this into account. This was done by performing the integral $\int_0^\beta 2\pi R^2 \sin\theta \cos^2\theta d\theta$ (see Appendix A). The muon flux per minute was divided by the solid angle (with $\cos^2\theta$ dependence) and plotted with respect to the separation between the PMTs. This data was fitted to a constant, which was found to be 14.0±0.6 muons/min/sr. Using the area of PMTB ($25 cm^2$), the constant can also be written as 0.56±0.02 muons/cm^2/min/sr.

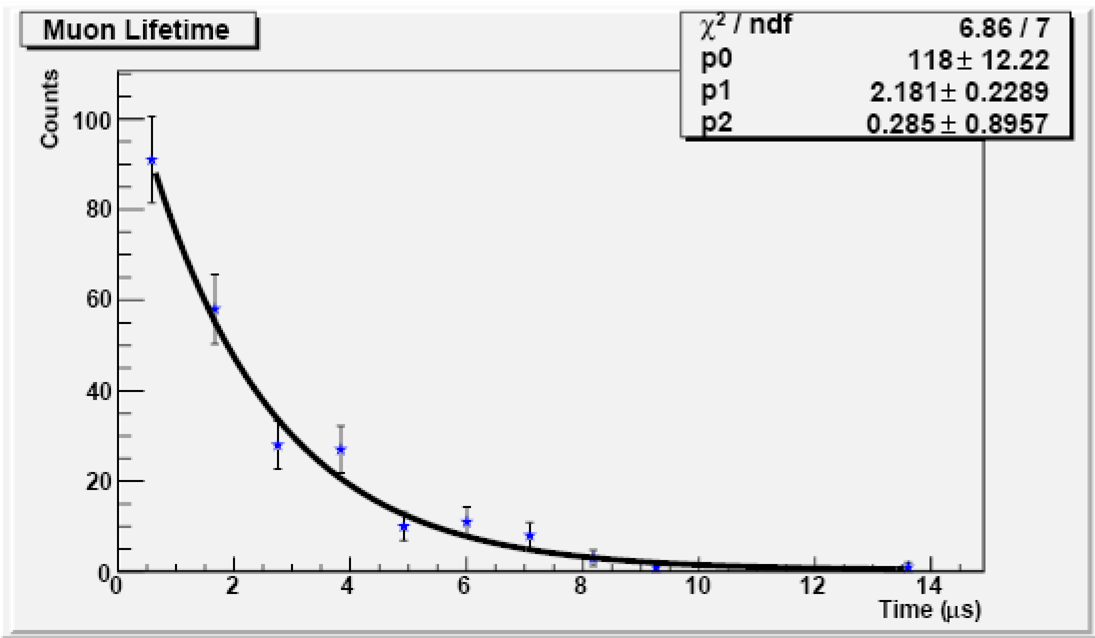

Figure 5: Graph of data for muon lifetime measurement, fit equation: $0.285 + 118e^{-t/2.181}$

Due to low data rates, the raw data from the TAC was rebinned and fitted to an equation of the known form for the number of muon decays as a function of time plus a constant background. The equation was found to be $0.285 + 118e^{-t/2.181}$.

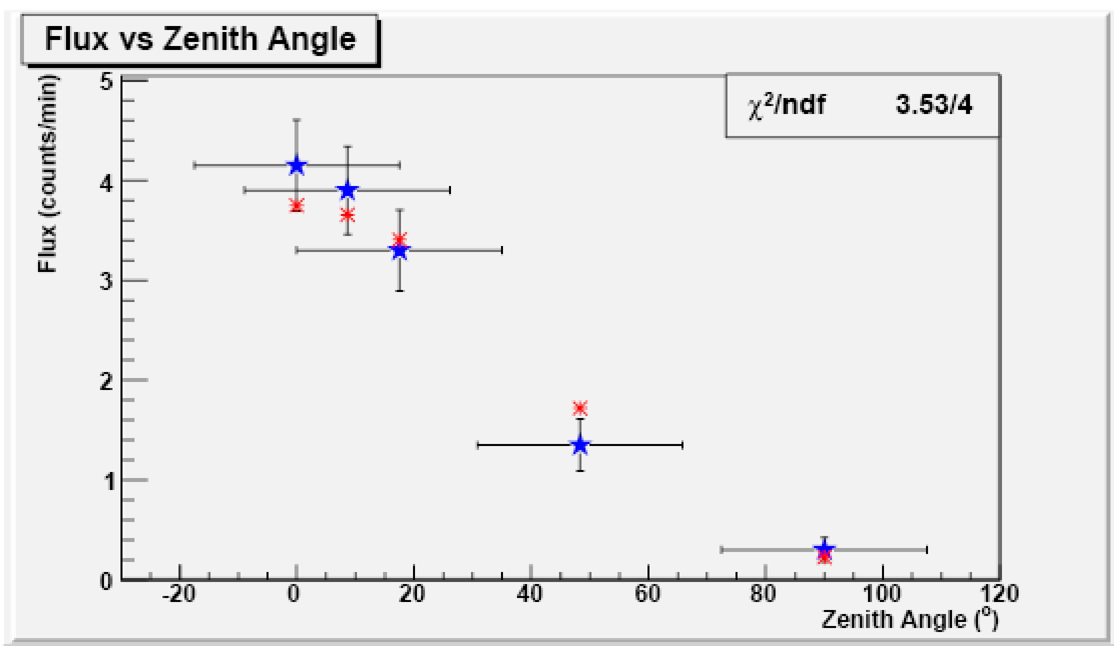

Figure 6: Plot of data for zenith angle measurement. Blue stars denote experimental data and red stars denote the theoretical measurements. χ^2/ndf value was calculated to be 3.53/4.

The flux of muons is known to have a zenith angle dependence of $\cos^2\theta$. Because of the finite dimensions of the PMT detectors, the zenith angle measurements represent the number of counts over the integral of $\cos^2\theta$ for a range of angles (rather than the number of counts at a particular zenith angle). Figure 6, which shows the data for the zenith angle measurements, accounts for this. The blue stars represent the experimental data. The error bars in the X-direction indicate the range of zenith angles from which muons could be accepted when taking data. The error bars in the Y-direction are are due to counting statistics and are equal to $\sqrt{n}$. The red stars represent the theoretical expectation for the flux integrated over the same range of zenith angles. This calculation uses a normalization of 3.75 counts/min over the range of angles centered at 0^{o}. The normalization was determined by a least squares method of fitting. The χ^2 value is calculated from the equation: $\chi^2 = \sum \frac{(N - N_{obs})^2}{\delta_{N_{obs}}{}^2}$, where N is the theoretical flux, N_{obs} is the experimentally observed flux, and $\delta_{Nobs}$is the error in the experimentally observed flux. The χ^2/ndf value was calculated to be 3.53/4.

VI. Conclusion

The data for the flux at various separations of PMTA and PMTB divided by the solid angle should be a constant. The graph in Figure 4 shows that the fitted constant falls within the range of the data, as indicated by the low χ^2/ndf value. The fitted constant, 0.56±0.02 muons/cm^2/min/sr, is lower than the theoretical rate of muons at sea-level (1 muons/cm^2/min/sr). Because the lab is located on the second floor of Watanabe Hall, there are a few levels of solid concrete, people, and equipment that the muons must pass through. This greatly reduces the rate of muons in the lab and is reflected in the data.

The data for the muon lifetime measurement (Figure 5) gives a lifetime of 2.18±0.23μs. This agrees with the expected value of 2.197μs.

The data for the flux at various zenith angles agrees fairly well with theory. In Figure 6, all but one value for the theoretical flux falls within the error of the experimentally determined flux, as indicated by the χ^2/ndf value of 3.53/4.

VII. References

(1) "Muon." *Wikipedia.* http://en.wikipedia.org/wiki/Muon.

(2) Peters, Michael W. "Solid Angle Subtended by a Flat Square Counter.

VIII. Appendix A: Calculation of Solid Angle

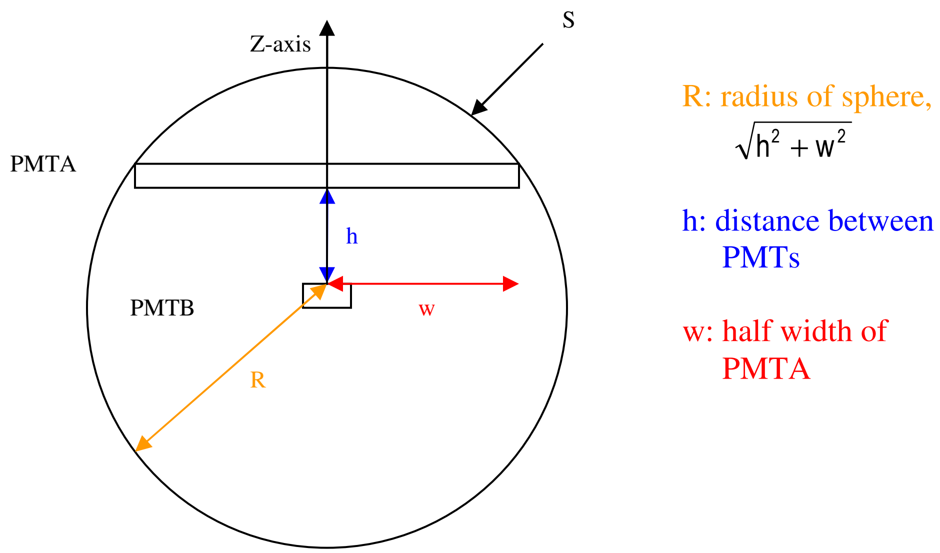

Figure 7: Calculation of Solid Angle

From Figure 7, the area S, which corresponds to the upper surface of the sphere, can be found by the integral $\int_0^\beta 2\pi R^2 \sin\theta d\theta$, where θ is the zenith angle and β is the zenith angle found by the equation $\beta = \tan^{-1}(w/h)$. Thus, the solid angle corresponding to S can be found by $\frac{4\pi}{4\pi R^2}\int_0^\beta 2\pi R^2 \sin\theta d\theta$. The zenith angle dependence ($\cos^2\theta$) of the muon flux can be taken into account by inserting $\cos^2\theta$ into the integral. Thus, the solid angle (with the zenith angle dependence of muon flux taken into account) corresponding to S can be found by $\int_0^\beta 2\pi R^2 \sin\theta \cos^2\theta d\theta$. It should be noted that this equation estimates PMTA to be a circle of radius w and PMTB to be a point. Unfortunately, the exact calculation with a square for PMTA and a finite width for PMTB is too difficult.