γ-γ Angular Correlation Measurements

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I. Abstract

This experiment used two detectors (Figure 1) to determine the angular correlation between gamma rays emitted by ^{22}Na and ^{60}Co. One detector was fixed while the other was free to rotate. The signals from each detector were discriminated and sent into a coincidence module. The number of coincident signals was recorded with a scaler. The moveable detector was rotated and the relationship between the number of coincidences and the angle between the detectors was determined. The experiment was conducted for three radial separations (2.5cm, 5cm, 12.2cm) from the source to the detectors using the ^{22}Na source and for one radial separation (5cm) using the ^{60}Co source.

The data for ^{22}Na was fitted to a gaussian distribution. The experimentally determined widths of the gaussian distribution for a radial separation of 12.2cm agrees with theory. The widths for the other two radial separations do not agree with theory because they are smaller than the theoretical widths. The theoretical widths come from the geometry of overlapping circles. However, this only works for small angles. The anisotropy of ^{22}Na was calculated to be 256.7±101.0, 154.4±67.0, 78.8±40.4. The experimental values agree with the theoretical values, which are infinitely large.

The data for ^{60}Co was fitted to the known equation: $C(\theta) = A(1+a_1(\cos\theta)^2+a_2(\cos\theta)^4)$. The theoretical values for a_1 and a_2 are .125 and .042. These agree with the experimental values: a_1=.195±.083 and a_2=.055±.025. The anisotropy for ^{60}Co was calculated to be.39±.12, which does not agree with the theoretical value of .2. This may be due to an underestimation in the error for the anisotropy and systematic drift in the detector over time. (1)

II. Introduction

^{22}Na is a positron emitter. The ^{22}Na is wrapped in a thin sheet of copper. The slow positrons are captured in the copper, where they combine with the electrons in the copper to form positronium. After a very short time, the positronium decays into two .511 MeV gamma rays. Because the positronium was at rest in the copper, by conservation of momentum, theoretically the gamma rays must be emitted in opposite directions.

^{60}Co decays to ^{60}Ni in a series of decays with the emission of gamma rays. The first decay emits a 1.172 MeV gamma ray and the second decay emits a 1.333 MeV gamma ray. Since the lifetime of the intermediate state is on

the order of 10^{-12} seconds, the emitted gamma rays are nearly coincident. Given the emission of one gamma ray, the second will be correlated to the first by a function of angle [1]

This experiment used ^{22}Na to check the calibrations of the apparatus. This was accomplished by checking the anisotropy and showing that the data followed a gaussian distribution. The anisotropy of ^{60}Co was also calculated.

III. Apparatus

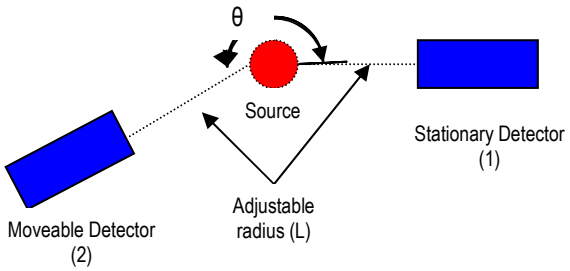

Figure 1: Apparatus used for angular correlation measurements. Detectors consist of a scintillator with a photomultiplier tube. Each PMT was operated at 1600V.

The apparatus shown in Figure 1 was used to take measurements. The detectors were plastic scintillators, 4cm in diameter, mounted on RCA 6655 photomultiplier tubes. The two detectors had variable radii and the moveable detector rotated continuously from 90˚ to 270˚.

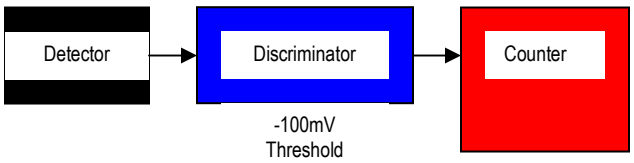

Figure 2: Experimental setup used to determine plateau curve and singles rate in each Detector.

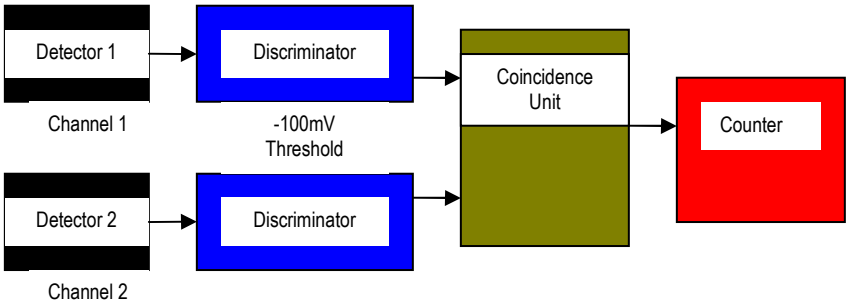

Figure 3: Experimental setup used to make angular correlation measurements.

The experimental setups shown in Figure 2 and Figure 3 were used in the experiment. Figure 3 shows that the output of each detector was put into a discriminator with a threshold of -100mV. The outputs of both discriminators

were then put into a coincidence unit, which output a signal to a counter capable of a handling peak rate of 10^5 reads per second. All cables were 50Ω coaxial cables.

IV. Procedure

Using the setup in Figure 2 and ^{22}Na as the source, a plateau curve for each scintillator was constructed. The optimum operating voltage was determined to be 1600V for both scintillators. With the detectors 180˚, the singles rate in each detector was determined by measuring the number of counts in 10 seconds.

Modifying the setup in Figure 3, a variable delay was inserted in channel 2 between the detector and the discriminator. The number of coincidences was determined as a function of the delay in channel 2. The accidental rate was then measured by placing the two detectors at 90˚ and putting a long fixed delay in channel 2 (~50nS).

Using the setup in Figure 3, the number of coincidences in 60 seconds was determined as a function of the angle between the detectors. This measurement was performed for different distances between the detectors and the source (which will be denoted as L). L was set to 2.5 cm, 5 cm, and 12.2 cm.

^{60}Co was used as the source for the setup in Figure 3 to determine the number of coincidences in 60 seconds as a function of the angle between the detectors. L was set to 5cm.

V. Calculations

The rate of accidental coincidences is determined by the equation: $R_A=R_1R_2\Delta t$, where R_1 and R_2 are the singles rate in detectors 1 and 2 respectively and Δt is determined by the width of the delay curve. It was found that R_1 was 121counts/s, R_2 was 84.4counts/s, and the width of the delay curve was 20nS. Thus, R_A = (121)(84.4)(20) = 2.0(10^{-4}) counts^2/s [1].

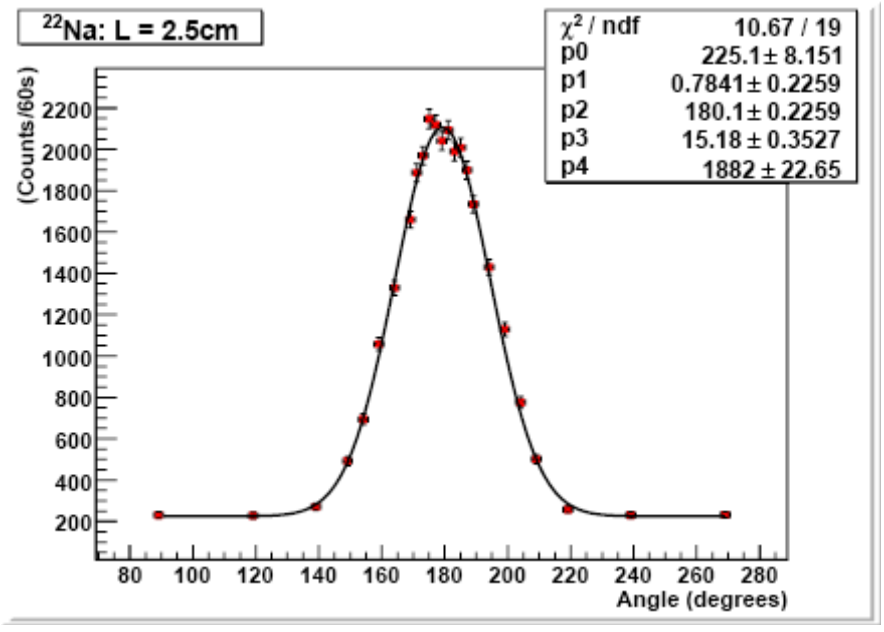

Figure 4: Plot of ^{22}Na data with L = 2.5 cm fit to gaussian. Fit equation: $1882e^{-\frac{1}{2}\left(\frac{\theta-180.1}{15.18}\right)^2}+225.1$

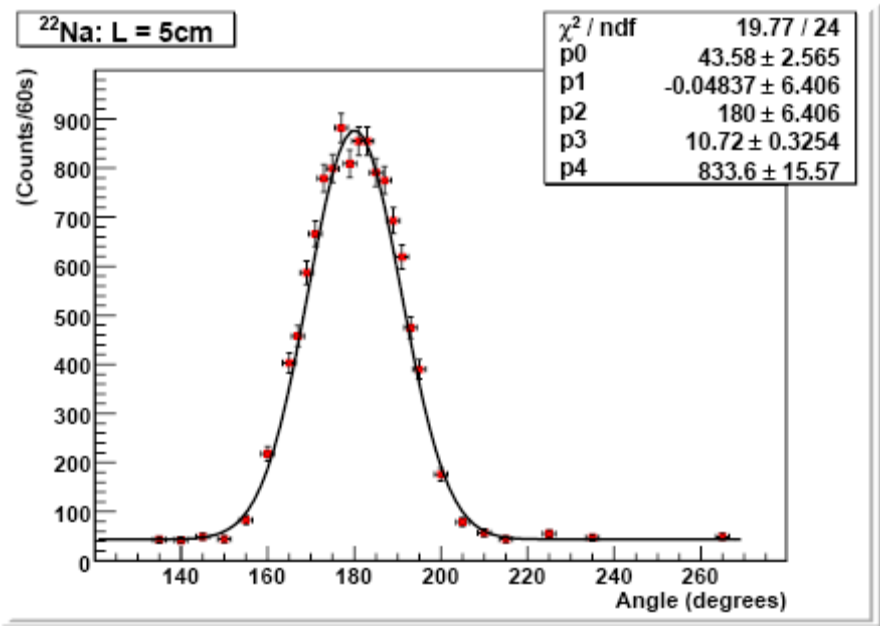

Figure 5: Plot of ^{22}Na data with L = 5cm fit to gaussian. Fit equation: $834e^{-\frac{1}{2}(\frac{\theta-180}{10.72})^2} + 43.6$

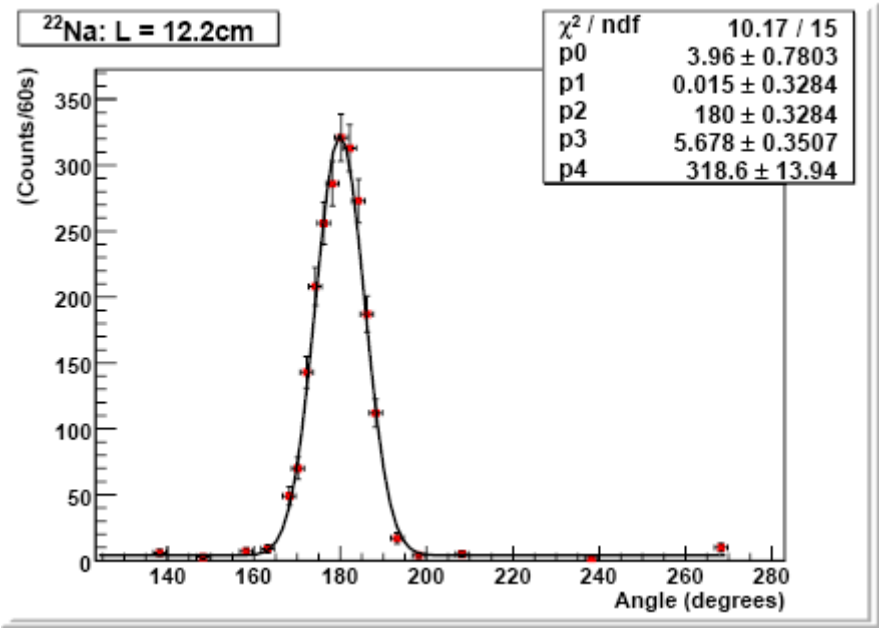

Figure 6: Plot of ^{22}Na data with L = 12.2cm fit to gaussian. Fit equation: $319e^{-\frac{1}{2}(\frac{\theta-180}{5.68})^2} + 4.0$

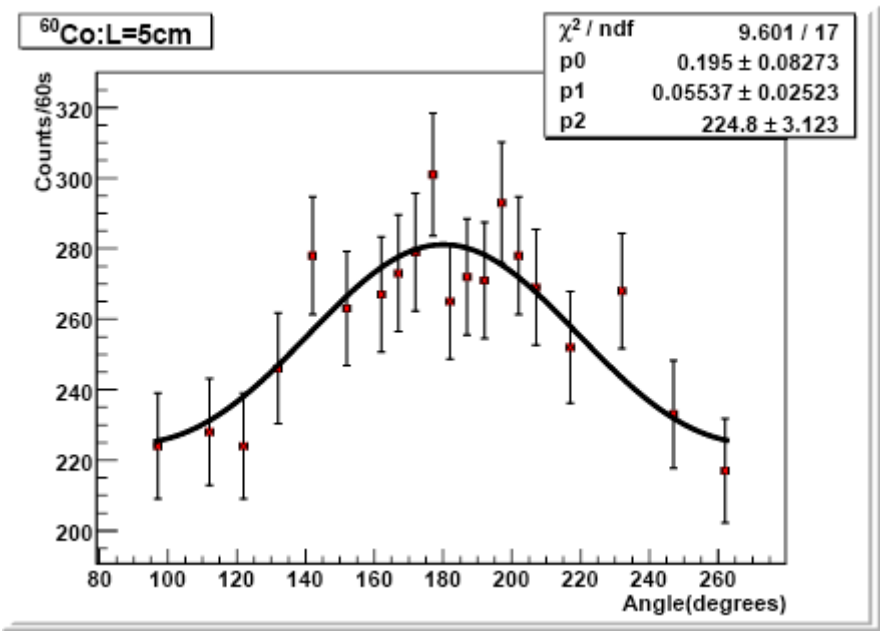

Figure 7: Plot of ^{60}Co data with L = 5cm. Fit equation: $224.8(1+.195(\cos\theta)^2+.055(\cos\theta)^4)$

Figures 4-7 show plots of the data for ^{22}Na and ^{60}Co. The figures display the number of counts as a function of angle. The plots for ^{22}Na (Figures 4-6) have gaussian fits. The flat offset in the plots is much higher than the rate of accidentals. This is due to the third photon, which is not correlated with angle. The coincidence rate of ^{60}Co is fit to the known equation: $C(\theta) = A(1+a_1(\cos\theta)^2+a_2(\cos\theta)^4)$, where A, a_1, and a_2 are constants [1].

The coincidence rate for ^{22}Na is predicted by the geometry of the overlapping area between the detectors. Because the detectors were circular, the area and thus the coincidence rate is given by $C(\theta) = 2R^2\sin^{-1}\sqrt{1-(\frac{L\sin\theta}{2R})^2} - L\,|\sin\theta|\sqrt{R^2-(\frac{L\sin\theta}{2})^2}$, where R is the radius of the scintillator (2cm). The theoretical width of the gaussian distribution is the angle where $e^{-1/2}C(0)=C(\theta)$. The calculated values for the theoretical widths of the ^{22}Na distributions are in Table 1.

The anisotropy is defined as $\alpha = \frac{C(180°) - C(90°)}{C(90°)}$, where C(θ) is number of counts as a function of angle [1].

The error in α can be found by $\Delta\alpha = \sqrt{(\frac{\Delta C(180°)}{C(90°)})^2 + (\frac{C(180°)\Delta C(90°)}{C(90°)^2})^2}$, where ΔC(θ) is the error in C(θ). The calculated values for α are in Table 1.

Source	L (cm)	Calculated α	Theoretical α		Calculated Width of Gaussian (from fit to data)	Theoretical Width (from overlapping circles)
^{22}Na	2.5	256.7±101.0	∞		15.18˚±.35˚	30.37˚
^{22}Na	5	154.4±67.0	∞		10.72˚±.33˚	14.32˚
^{22}Na	12.2	78.8±40.4	∞		5.68˚±.35˚	5.96˚
^{60}Co	5	.39±.12	.2		---	---

Table 1: Calculated and Theoretical Values for α and Width of Gaussian Distribution

VI. Conclusion

The observed rate of accidental coincidences was 0 counts^2/s, which agrees with the calculated value of 2.0(10^{-4}) counts^2/s. As expected, the gaussian fits for the ^{22}Na source at different L became smaller as L increased. This is due the fact that the overlapping area between the detectors decreases more rapidly as the distance between them decreases.

Because the gamma rays emitted from ^{22}Na are exactly opposite in direction, the theoretical value for C(θ) is δ(π-θ). Thus, $\alpha = \frac{\delta(\pi - 180°) - \delta(\pi - 90°)}{\delta(\pi - 90°)}$ is theoretically infinite. The experimental values for α of the ^{22}Na source are large enough to agree with theory.

The width of the ^{22}Na data for L=12.2cm agrees with the theoretical value. However, the other widths (for L=2.5cm and L=5cm) do not agree with theory. This may be because the equation given for the coincidence rate based on the geometry assumes no rotation of the scintillator surface. The equation assumes that the moveable scintillator surface remains parallel to the stationary scintillator surface (while moving from side to side). This assumption only works for small angles. Indeed, The theoretical value for the width of ^{22}Na with L=12.2 cm is small (5.96˚) and the data agrees with it.

Although the theoretical value for α for ^{60}Co is about .2, the calculated value of .39±.12 is not close to this. The theoretical value does not fall within the error of the calculated value. This may be from underestimation of the error in the calculated α. The error was calculated with the equation for $\Delta\alpha$ as stated above, with $\Delta C(\theta) = \sqrt{C(\theta)}$. This may not have been sufficient because of systematic drift in the detector over time. It can be noted that ^{22}Na has a much stronger angular dependence than ^{60}Co. The distributions for the ^{22}Na data are narrower and the anisotropy is much higher.

The ^{60}Co data is fit to the equation: $A(1+a_1(\cos\theta)^2+a_2(\cos\theta)^4)$. Theoretically, A is a normalization constant, a_1=.125, and a_2=.042. The experimentally determined values are a_1=.195±.083 and a_2=.055±.025. These values agree with theory.

VII. References

1. Melissinos, Adrian C. and Napolitano, Jim. *Experiments in Modern Phyiscs*. 2nd edition. Pg. 409-21 (2003).